The basic principles of geometrization of the quantum mechanics

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Abstract

The basic principles of the quantum mechanics in the K-field formalism [1] are stated in the paper. K-field formalism arises from geometric generalization of de Broglie postulate. So, the quantum theory equations (including well-known Schrödinger, Klein-Gordon and quadratic Dirac equations) are obtained as the free wave equations on a manifold metrizing force interactions of particles.

In this paper, describing wave properties of particles we will restricted ourself with construction special geometric formulation of force interactions.

n1. The de Broglie hypothesis initial sense is that it is possible to spread the formulas describing a photon behaviour to mass particles too. Such a view on the de Broglie theory allows one to use it for a formal description of a microparticles behaviour in terms of a photon behaviour formal description.

Such a conception of the de Broglie hypothesis we shall accept as a basis of the quantum theory formalism.

n2. To present correlation of photon and microparticle description more obviously let's make a table (see tab. 1).

Obviously by virtue of such essential difference in description of photons and microparticles the direct prolongation of the equations for photons on mass particles is impossible.

Let's look at this problem a little differently. Let's consider motion of particles from an isolated observer's view point.

Photons move along an isotropic geodesic lines of the Minkowski four-space V_4. Hence, it is necessary to build a four-space in which mass particles should move (relatively to an isolated observer) along an isotropic geodesic lines of this space.

Table 1

photons	mass particles
move uniform rectilinearly with the velocity c relatively to any inertial frame	move along any trajectories with any velocities (less than c) relatively to any inertial frame
the wave properties are described by the equation of d'Alember $$\Box A^\mu \,=\, 0$$ for an electromagnetic field four-potential A^μ	the wave properties are described by the Klein-Gordon equation $$[(E-V)^2 + \hbar^2 c^2 \triangle - m^2 c^2]\Psi \,=\, 0$$ for a particle Ψ-function

Let's now collect all the isotropic surfaces of separate observers in one four-manifold ${}^kV_4$. We shall obtain the manifold ${}^kV_4$ in which mass particles will move (relatively to any observer) along isotropic geodesic lines of this manifold for any motion.

Hence, description of a mass particle on the manifold ${}^kV_4$ and description of a photon in the Minkowski four-space V_4 becomes equivalent. And so, we can formulate the de Broglie hypothesis as follows:

> to describe wave properties of particles it is necessary
>
> - to build the manifold ${}^kV_4$ in which mass particles move along geodesic lines of this manifold for any motions;
> - to build an operator the similar to that d'Alember on the manifold ${}^kV_4$ (so-called the de Rham operator ${}^{(k)}\triangle$);
>
> then the equation
>
> $$({}^{(k)}\triangle k)_\mu \,=\, 0\,, \tag{1}$$
>
> where k_μ is the K-field potential should describe the wave properties of mass particles.

Let's sum up everything, mentioned above, in a table (see tab. 2)

Table 2

photons	mass particles
move along isotropic geodesic lines of the Minkowski four-space V_4	move along isotropic geodesic lines of the manifold ${}^kV_4$
the wave properties are described by the equation of d'Alember $$\Box A^\mu \,=\, 0$$ for the electromagnetic field four-potential A^μ	the wave properties are described by the equation $$({}^{(k)}\triangle k)_\mu \,=\, 0$$ for the K-field potential k_μ, where ${}^{(k)}\triangle$ is the de Rahm operator and k_μ is linear form

n3. In this paper, describing wave properties of particles we will restricted ourself with construction special geometric formulation of force interactions.

Geometrization of an interaction consists in finding a metric space in which the test particle trajectories are geodesic lines [2]. This is the starting point of Einstein concept of geometrization.

An interesting method of metrization of arbitrary force interactions corresponding to this concept was presented in [3]. In this method of metrization, the test particles move along geodesic lines. However, the force fields are related with the components of the connection tortion tensor of a pseudo-Euclidean space. In this sense, the metrization of force interactions presented in [3] does not correspond to the Einstein concept because the metric properties of the space do not depend on force fields.

So, we shall consider a metric statement of force interactions in which, as in [3], the test particle motion equations represent a special form of Newton's second law in four-dimensional form but the metric tensor and physical fields are interdependent.

To avoid the problems connected with the distinction between the concepts of a reference frame and a coordinate system [3], different observers (i.e., reference frames) will be associated with different isotropic surfaces on the manifold ${}^kV_4$.

n4. The states of the test particles (of mass m and charge e) in potential fields will be called classical states. Correspondingly, all the characteristics of the particle describing its behavior in the classical state (trajectory, velocity, momentum, energy, etc.) will be called classical.

It should be emphasized that all classical characteristics should be measured relatively to one specific reference frame. Any inertial frame (IF) may be selected as that reference frame.

Let's consider some a four-dimensional space with the metric

$$ {}^{(k)}dS^2 \;=\; {}^{(k)}g_{oo}(x^i,t)c^2dt^2 \;+\; g_{ik}dx^idx^k \;, \tag{2} $$

where $(-g_{ik})$ is the metric tensor of the Euclidean space V_3.

Any classical trajectory $x^i = x^i(t)$ may be considered as a line defined by the equation ${}^{(k)}g_{oo}(x^i(t),t)c^2dt^2 = --\, g_{ik}dx^idx^k$. And so along the line

$$ {}^{(k)}g_{oo}(x^i(t),t) \;=\; v^iv_i/c^2 \;, \tag{3} $$

where v^i is the particle velocity measured relatively to the specified IF ($v_i = --\, g_{ik}v^k$).

Thus, each point p of the classical particle trajectory $x^i = x^i(t)$ in V_3 may be considered as a line lying on the isotropic surface ${}^kG_{o3} \subset {}^kV_4$ described by the equation ${}^{(k)}g_{oo}c^2dt^2 = --\, g_{ik}dx^idx^k$.

Hence, each point $p \in V_3$ may also be considered as a point of the isotropic surface ${}^kG_{o3}$ in ${}^kV_4$. That is an isotropic surface ${}^kG_{o3} \subset {}^kV_4$ may be constructed at points of space V_3. By changing the values of the initial parameters, a set of points covering the whole of ${}^kG_{o3}$ may be obtained. And by transiting from one reference frame to another, a set of surfaces ${}^kG_{o3}$ covering the whole of ${}^kV_4$ may be obtained. That is an imbedding [2] (enclosure in a space of higher dimensionality) may be constructed.

n5. According to Eq.(2), the method of enclosure described in Sec.5 should have the distinctive property. Namely, the geometry of the enclosing space ${}^kV_4$ should have no influence on the geometric properties of the enclosed space V_3 (should not change the metric tensor g_{ik}). In other words, the imbedding must occur at those points of ${}^kV_4$ at which the external curvature of the enclosed surface is zero.

So, then it follows from the Gauss–Vaingarten equations (see [4], for example), the absolute differential of the space ${}^kV_4$ (denoted by ${}^{(k)}\nabla(\cdots)$) is defined by the equation

$$ {}^{(k)}\nabla A^\mu \;=\; ({}^{(3)}\nabla_i A^\mu)dx^i \;+\; ({}^{(4)}\nabla_o A^\mu)dx^o \;. \tag{4} $$

Equation (4) may also be rewritten in the form

$$ {}^{(k)}\nabla A^\mu \;=\; {}^{(k)}DA^\mu \;+\; {}^{(k)}\Gamma^\mu_{\nu o}A^\nu dx^o \;, \tag{5} $$

where ${}^{(k)}DA^i = DA^i + {}^{(k)}S^i_{kl}A^kdx^l$ is the absolute differential of the Euclidean space V_3 (here ${}^{(k)}S^i_{kl} = S_{kl}{}^i - S_l{}^i{}_k - S_k{}^i{}_l$ and $S_{kl}{}^i$ is the tortion tensor) and ${}^{(k)}\Gamma^\mu_{\nu o}$ is the connection of the space ${}^kV_4$.

n6. To obtain a more detailed description, the definition of the absolute differential ${}^{(k)}\nabla(\cdots)$ is written in standard form [2], [5]

$$ {}^{(k)}\nabla A^\mu \;=\; (\partial_\nu A^\mu \;+\; \Gamma^\mu_{\omega\nu}A^\omega)dx^\nu \;, \tag{6} $$

where $2\Gamma^{\mu}_{\omega\nu} = 2{}^{(k)}\Gamma^{\mu}_{\omega\nu} + Q^{\mu}_{\omega\nu}$, at that $Q^{\mu}_{\omega\nu} = {}^{(k)}g^{\mu\gamma}({}^{(k)}Q_{\omega\nu\gamma} + {}^{(k)}Q_{\nu\gamma\omega} - {}^{(k)}Q_{\gamma\omega\nu})$ and ${}^{(k)}Q_{\mu\nu\omega} = -{}^{(k)}\nabla_{\mu}\left({}^{(k)}g_{\nu\omega}\right)$; ${}^{(k)}\Gamma^{\mu}_{\omega\nu} = \left\{{\mu \atop \omega\nu}\right\} + {}^{(k)}S^{\mu}_{\omega\nu}$, where $\left\{{\mu \atop \omega\nu}\right\}$ is the Christoffel symbol and ${}^{(k)}S^{\mu}_{\omega\nu} = S_{\omega\nu}{}^{\mu} - S_{\nu}{}^{\mu}{}_{\omega} - S_{\omega}{}^{\mu}{}_{\nu}$ at that $S_{\omega\nu}{}^{\mu}$ is the tortion tensor.

If it is required that the definition in Eq.(6) coincide with that in Eq.(5), the result obtained is

$$2{}^{(k)}\Gamma^{i}_{oj}dx^{j} + Q^{i}_{o\omega}dx^{\omega} = Q^{i}_{j\omega}dx^{\omega} = 2{}^{(k)}\Gamma^{o}_{\mu j}dx^{j} + Q^{o}_{\mu\omega}dx^{\omega} = 0\,, \qquad (7)$$

which must be satisfied if the imbedding described in Secs.5 and 6 is possible.

It may readily be demonstrated that the absolute differential ${}^{(k)}\nabla(\cdots)$ of space ${}^{k}V_4$ defined by Eq.(5) describes a nonmetric transfer in ${}^{k}V_4$. In fact

$${}^{(k)}Q_{ooo} = 2{}^{(k)}g_{oo}{}^{(k)}S^{o}_{oo}\,, \quad {}^{(k)}Q_{ioo} = -\partial_{i}{}^{(k)}g_{oo}\,. \qquad (8)$$

The remaining ${}^{(k)}Q_{\mu\nu\omega} = 0$. As a result the equations (7) take the form

$$\begin{aligned} {}^{(k)}S^{o}_{oj}dx^{j} &= -\left\{{o \atop oj}\right\}dx^{j} - 2{}^{(k)}S^{o}_{oo}dx^{o}\,, \\ {}^{(k)}S^{o}_{ij}dx^{j} &= -\left\{{o \atop ij}\right\}dx^{j} + \left\{{o \atop io}\right\}dx^{o}\,, \\ {}^{(k)}S^{i}_{oj}dx^{j} &= -\left\{{i \atop oj}\right\}dx^{j} + \left\{{i \atop oo}\right\}dx^{o}\,, \end{aligned} \qquad (9)$$

Hence it is clear that the tortion $S_{\omega\nu}{}^{\mu}$ is nonzero.

Thus, the imbedding described in Sec.2 generates in ${}^{k}V_4$ a geometry with tortion and a nonzero covariant derivative of the metric tensor.

n7. The test particle motion equations are now considered. It is desirable for these equations to coincide with the geodesic equations in ${}^{k}V_4$. Then these equations should take the form

$$Dp^{\mu} = -{}^{(k)}\Gamma^{\mu}_{\nu o}p^{o}dx^{\nu}\,, \quad (p^{\mu} = m dx^{\mu}/d\tau)\,. \qquad (10)$$

Taking this into account, the condition $dx_i dp^i = dx^o dp^o$ leads to the equation

$${}^{(k)}S^{j}_{\nu o}dx^{\nu}dx_{j} = \left(\left\{{o \atop \nu o}\right\} + {}^{(k)}S^{o}_{\nu o}\right)dx^{\nu}dx^{o} - \left\{{j \atop \nu o}\right\}dx^{\nu}dx_{j}\,, \qquad (11)$$

which, together with Eq.(9), describes all the nonzero components of ${}^{(k)}S^{\mu}_{\omega\nu}$.

Further, it is readily evident that, if the components ${}^{(k)}S^{o}_{\nu o}$ are chosen in the form

$${}^{(k)}S^{o}_{\nu o} = [\partial_{\nu}\ln((1 - {}^{(k)}g_{oo})/{}^{(k)}g_{oo})]/2\,, \qquad (12)$$

the four-momentum p^{o} component is found to be

$$p^{o} = C_1(1 - {}^{(k)}g_{oo})^{-1/2}\,, \qquad (13)$$

where $C_1 = const$. Assuming that $C_1 = mc$, it is found that $d\tau = (1 - {}^{(k)}g_{oo})^{-1/2}dt$.

Hence, Eqs.(11) and (12) are the necessary and sufficient conditions for the motion equations (10) to be noncontradictory.

Thus, the classical particle trajectories in the potential fields specified with respect to a definite IF may be represented as geodesic lines lying on isotropic surfaces of some configurational space ${}^{k}V_4$ the connection of which has tortion, while the transference is nonmetric. The geometry of the space ${}^{k}V_4$ has the distinctive property that the magnitude of the nonmetricity of the transfer and the tortion are determined by specifying the metric coefficient ${}^{(k)}g_{oo}$ under the condition that the mixed components ${}^{(k)}g_{oi} \equiv 0$.

Referenses

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